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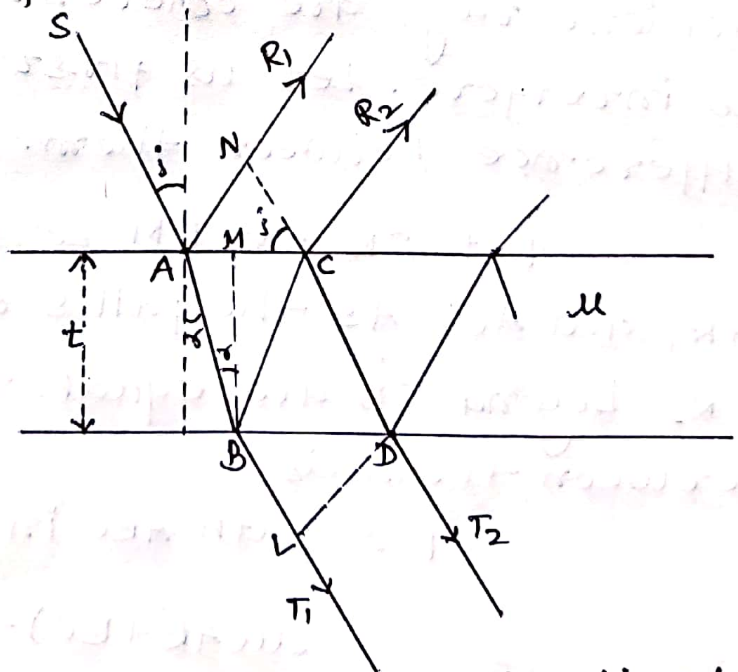
B.Sc (Part-II) Hons

Interference in thin films and show that with monochromatic light, the interference patterns of reflected light and transmitted light are complementry.

When a film of oil spread over the surface of water is viewed by white light reflected from it, brilliant colours are often seen. A similar colour phenomenon is obtained from soap bubbles or from thin glass plates. These colours arise due to the interference between the light waves reflected from the upper and lower surfaces of the film, or the plate.



Let a ray of monochromatic light SA be incident at an angle i on a parallel-sides transparent thin film of thickness t and refrective index μ. At A it is partly reflected along AR_1 and partly refrected along AB at angle r. At B it is again partly reflected along BC and partly reflected along BT_1. Similar reflections and refrections occur at C, D, --- etc. Thus we get a set of parallel reflected rays AR_1, CR_1 ---- etc. and a set of parallel trans-mitted rays BT_1, DT_2, --- etc. Let us consider the reflected rays only.

At each of the points A, B, C, D --- only a small part of light is reflected, the rest is refracted. Therefore the ray AR_1 and CR_2, each having undergone one reflection, have almost equal intensities. The rest have rapidly decreasing intensities and can be ignored. The Ray AR_1 and CR_2 being derived from the same incident ray, are coherent and in a position to interfere. Let us first calculate the path difference between them.

Let CN and BM be perpendiculars to AR_1 and AC. As the paths of the rays AR_1 and CR_2 beyond CN are equal, the path difference between them is

$$P = \text{path ABC in film} - \text{path AN in Air}$$

$$= \mu(AB+BC) - AN \quad \text{——— (1)}$$

Now, $$AB = BC = \frac{BM}{\cos r} = \frac{t}{\cos r}$$

and $$AN = AC \sin i$$
$$= (AM + MC)\sin i$$

$$AN = (BM\tan r + BM\tan r)\sin i$$

$$= 2\,BM\tan r \cdot \sin i$$

$$= 2t\,\frac{\sin r}{\cos r} \cdot \sin i$$

$$= 2t\,\frac{\sin r}{\cos r}(\mu \sin r) \qquad (\because \sin i = \mu \sin r)$$

$$\therefore AN = 2\mu t\,\frac{\sin^2 r}{\cos r}$$

Substituting these values of AB, BC and AN in equation (1)

$$p = \mu\left(\frac{t}{\cos r} + \frac{t}{\cos r}\right) - 2\mu t\,\frac{\sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r}\left(1 - \sin^2 r\right)$$

$$= \frac{2\mu t}{\cos r} \cdot \cos^2 r$$

$$p = 2\mu t \cos r$$

The ray AR_1, having suffered a reflection at the surface of a denser medium, undergoes a phase changes of π. which is equivalent to path difference of $\frac{\lambda}{2}$. Hence the effective path difference between AR_1 and CR_2 is

$$2\mu t\cos r - \frac{\lambda}{2}$$

Condition of Maxima and Minima in Reflected Light:-

The two rays will reinforce each other if the path difference between them is integral multiple of λ i.e. when

$$2\mu t\cos r - \frac{\lambda}{2} = n\lambda \qquad (n = 0, 1, 2, \ldots)$$

$$\text{or } 2\mu t\cos r = n\lambda + \frac{\lambda}{2}$$

or, $2\mu t \cos r = (2n+1)\frac{\lambda}{2}$ (Condition of Maxima) — ①

When this condition is satisfied, the film will appear bright in the reflected light.

Again the two rays will destroy each other if the path difference between them is an odd multiple of $\lambda/2$ i.e. when

$$2\mu t \cos r - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2} \quad (n = 1, 2, 3, ---)$$

$$2\mu t \cos r = (2n-1)\frac{\lambda}{2} + \frac{\lambda}{2}$$

$$2\mu t \cos r = (2n-1+1)\frac{\lambda}{2}$$

or. $2\mu t \cos r = 2n \cdot \frac{\lambda}{2}$

or, $2\mu t \cos r = n\lambda$ (Condition of Minima) ②

Under this condition, the film will appear dark in the reflected light.

Path difference in Transmitted Light! — The path difference between the transmitted rays BT_1 and DT_2 is similarly given by

$$P = \mu(BC + CD) - BL$$

$$= 2\mu t \cos r \text{ —— ⑤}$$

In this case there is no phase change due to reflection B or C because in either case the light is travelling from denser to rarer medium. Hence the effictive path difference between BT_1 and DT_2 is also $2\mu t \cos r$.

Condition of Maxima and Minima in Transmitted Light! —

The two rays BT_1 and DT_2 reinforce each other if

$$2\mu t \cos r = n\lambda \text{ —— ③}$$

When $n = 1, 2, 3, --$ the film will therefore appear bright in the transmitted light.

Again the two rays will destory each other if

$$2\mu t \cos r = (2n+1)\lambda/2 \text{ —— ④}$$

where $n = 0, 1, 2, ---$ and the film will appear dark in the transmitted light.

A Comparision of equation (1), (2), (3) and (4) shows that the conditions for maxima and minima in the reflected light are just the reverse of those in transmitted.

Hence the appearances in the two cases are complementry to each other.